2.1 – Inductive Reasoning

**Defined Terms**

**Conjecture** – An unproven statement that is based on observations.

**Inductive Reasoning** – Finding a pattern in specific cases and then making a conjecture for the general case.

**Counterexample** – A specific case for which a conjecture is false.

**Example #1**

What is the next number in the sequence?

1, 1, 2, 3, 5, 8, 13, 21, 34, ___

Conjecture: after the first two 1’s, each number appears to be the sum of the two previous numbers (2 = 1+1, 3 = 1+2, etc…)

Inductive Reasoning: Using our conjecture, we predict that the next number in the sequence is $21 + 34 = 55$.

**Example #2**

Conjecture: adding two numbers together always results in an even number

Counterexample: $3 + 4 = 7$ (odd)

2.2 – Conditional Statements

A *conditional statement* is a logical statement that has two parts, a *hypothesis* and a *conclusion*. This type of statement can be written in *if-then* form.

The **hypothesis** of the statement is the part following the *if* and the **conclusion** is the part following the *then*.

**Example**

The statement, *All dogs are mammals* can be written in if-then form, as follows:

*If an animal is a dog, then it is a mammal.*

**Hypothesis:** an animal is a dog

**Conclusion:** the animal is a mammal

**Related Conditional Statements**

To write the *converse* of a conditional statement, exchange the hypothesis and conclusion of the original statement.

**Example**

Original statement: $a \rightarrow b$

*If it is raining, then I will get wet.*

Converse statement: $b \rightarrow a$

*If I am getting wet, then it is raining.*

To write the *inverse* of a conditional statement, negate the hypothesis and conclusion of the original statement.

**Example**

Original statement: $a \rightarrow b$

*If it is raining, then I will get wet.*

Inverse statement: $\sim a \rightarrow \sim b$

*If it is not raining, then I will not get wet.*

To write the *contrapositive* of a conditional statement, negate AND exchange the hypothesis and conclusion of the original.

**Example**

Original statement: $a \rightarrow b$

*If it is raining, then I will get wet.*

Contrapositive statement: $\sim b \rightarrow \sim a$

*If I am not getting wet, then it is not raining.*

**Equivalent Statements**

- The *contrapositive* always has an equivalent meaning to the original.
- The *converse* always has an equivalent meaning to the *inverse*.

**Perpendicular Lines**

If two lines intersect to form a right angle, then they are *perpendicular lines.*
2.3 – Deductive Reasoning

**Deductive Reasoning**

Deductive reasoning uses facts, definitions, accepted properties and the laws of logic to form a logical argument – much like what you see in mystery movies or television shows such as *Sherlock Holmes* or *CSI*.

**Laws of Logic**

**Law of Detachment**
If the hypothesis of a true conditional statement is true, then the conclusion of the statement is also true.

**Example**
Assume the following to be true:

*If it rains, then you will get wet.*

**Given** – It is raining
**Deduction** – you will get wet.

**Law of Syllogism**

<table>
<thead>
<tr>
<th>Form</th>
<th>Symbolization</th>
</tr>
</thead>
<tbody>
<tr>
<td>If a, then b</td>
<td>( a \rightarrow b )</td>
</tr>
<tr>
<td>If b, then c</td>
<td>( b \rightarrow c )</td>
</tr>
<tr>
<td>( \therefore ) If a, then c</td>
<td>( \therefore a \rightarrow c )</td>
</tr>
</tbody>
</table>

If the first two statements are true, then the third statement must be true as well.

**Example**
Assume the following to be true:

*If it rains, then you will get wet.*  
*If you get wet, then you will cough.*

Then the following conclusion can be made by deductive logic:

*If it rains, then you will cough.*

2.4 – Postulates and Diagrams

**Postulate 5** – Through any two points there exists exactly one line.

**Postulate 6** – A line contains at least two points.

**Postulate 7** – If two lines intersect, then their intersection is exactly one point.

**Postulate 8** – Through any three noncollinear points there exists exactly one plane.

**Postulate 9** – A plane contains at least three noncollinear points.

**Postulate 10** – If two points lie in a plane, then the line containing them lies in the plane.

**Postulate 11** – If two planes intersect, then their intersection is a line.
### 2.5 – Properties from Algebra

#### Algebraic Properties of Equality

**Addition Property of Equality**
If \( a = b \), then \( a + c = b + c \).

**Subtraction Property of Equality**
If \( a = b \), then \( a - c = b - c \).

**Multiplication Property of Equality**
If \( a = b \), then \( ac = bc \).

**Division Property of Equality**
If \( a = b \) and \( c \neq 0 \), then \( \frac{a}{c} = \frac{b}{c} \).

**Substitution Property of Equality**
If \( a = b \), then you may replace \( b \) with \( a \) in any equation or expression (and vice-versa).

**Distributive Property**
\( a(b + c) = ab + ac \)

**Reflexive Property of Equality**
For any real number \( a \), \( a = a \)

**Symmetric Property of Equality**
For any real numbers \( a \) and \( b \), if \( a = b \), then \( b = a \)

**Transitive Property of Equality**
For any real numbers \( a \), \( b \) and \( c \), if \( a = b \) and \( b = c \), then \( a = c \)

### 2.6 – Segment and Angle Proofs

#### Congruence of Segments
Segment congruence is reflexive, symmetric and transitive.

<table>
<thead>
<tr>
<th>Property</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Reflexive</td>
<td>( AB \cong AB )</td>
</tr>
<tr>
<td>Symmetric</td>
<td>If ( AB \cong CD ), then ( CD \cong AB )</td>
</tr>
<tr>
<td>Transitive</td>
<td>If ( AB \cong CD ) and ( CD \cong EF ), then ( AB \cong EF )</td>
</tr>
</tbody>
</table>

#### Congruent of Angles
Angle congruence is reflexive, symmetric and transitive.

<table>
<thead>
<tr>
<th>Property</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Reflexive</td>
<td>( \angle A \cong \angle A )</td>
</tr>
<tr>
<td>Symmetric</td>
<td>If ( \angle A \cong \angle B ), then ( \angle B \cong \angle A )</td>
</tr>
<tr>
<td>Transitive</td>
<td>If ( \angle A \cong \angle B ) and ( \angle B \cong \angle C ), then ( \angle A \cong \angle C )</td>
</tr>
</tbody>
</table>

### 2.7 – Angle Pair Relationships

#### Right Angles Congruence Theorem
All right angles are congruent.

#### Congruent Supplements Theorem
If two angles are supplementary to the same angle, those angles are congruent.

#### Congruent Complements Theorem
If two angles are complementary to the same angle, those angles are congruent.

#### Linear Pair Postulate
If two angles form a linear pair, then they are supplementary.

#### Vertical Angles Congruence Theorem
Vertical angles are congruent.